



Oxford Cambridge and RSA

**Friday 24 May 2024 – Afternoon**

**AS Level Further Mathematics B (MEI)**

**Y413/01 Modelling with Algorithms**

**Time allowed: 1 hour 15 minutes**

**You must have:**

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

**QP**



**INSTRUCTIONS**

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined page at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

**INFORMATION**

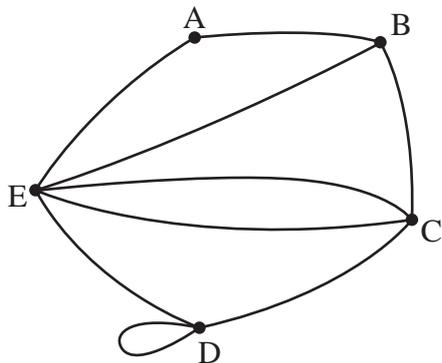
- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [ ].
- This document has **8** pages.

**ADVICE**

- Read each question carefully before you start your answer.

1 **Fig. 1** shows a graph.

**Fig. 1**



- (a) (i) State the order of node B. [1]
- (ii) State the order of node D. [1]
- (b) Complete the incidence matrix for the graph in **Fig. 1** in the Printed Answer Booklet. [1]
- (c) Explain why the graph in **Fig. 1** is not simple. [1]
- (d) Write down the number of arcs that need to be removed from the graph in **Fig. 1** for it to be a tree. [1]

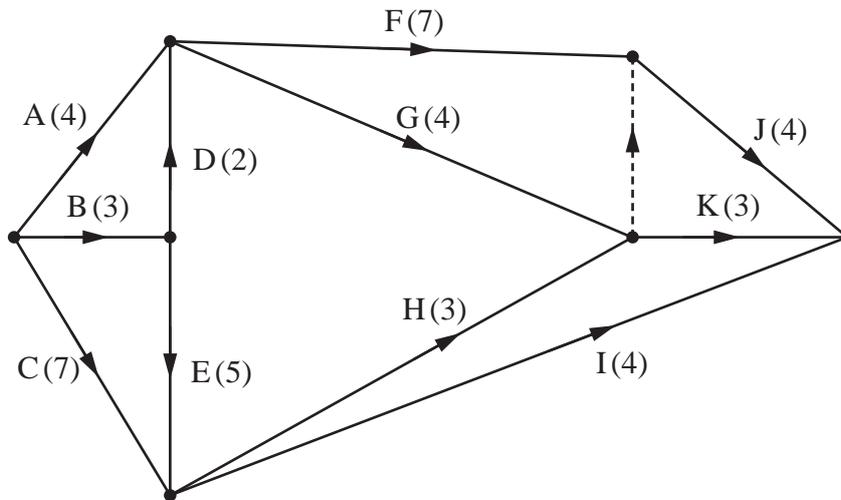
## 3

2 The list below shows the sizes of nine items.

31      13      12      17      25      18      11      8      15

- (a) Show the result of applying the first fit algorithm to pack items with the sizes listed above into bins that have a capacity of 50. [1]
- (b) Apply the quick sort algorithm to sort the list of numbers above into **ascending** order. You should use the **first** value as the pivot for each sublist. [3]
- (c) Show the result of applying the first fit decreasing algorithm to pack items with the sizes listed above into bins that have a capacity of 50. [2]
- (d) By finding an optimal packing using a full bin strategy, explain why both first fit and first fit decreasing are examples of heuristic algorithms. [2]

- 3 The diagram shows an activity network for a project. The number in brackets show the duration of these activities in hours.



- (a) Complete the table in the Printed Answer Booklet to show the immediate predecessors for each activity. [2]
- (b) (i) Carry out a forward pass and a backward pass through the activity network, showing the early event time and the late event time at each vertex of the network. [3]
- (ii) State the minimum project completion time. [1]
- (iii) State the critical activities of the project. [1]
- (c) Calculate the independent float for activity I. [1]

Each activity requires one person. When an activity is started it must be completed without interruption.

- (d) Use the diagram in the Printed Answer Booklet to show how **three** people can complete the project in the minimum time found in 3(b)(ii). Each column in the diagram represents 1 hour. For each person, write the letter of the activity they are doing in each box, or leave the box blank if the person is resting for that 1 hour. [2]

- 4 A student sets up the following initial tableau in **Fig. 4.1** to solve a maximisation LP problem in  $x$ ,  $y$ , and  $z$ .

**Fig. 4.1**

| $P$ | $x$ | $y$ | $z$ | $s_1$ | $s_2$ | $s_3$ | RHS |
|-----|-----|-----|-----|-------|-------|-------|-----|
| 1   | -2  | 3   | -1  | 0     | 0     | 0     | 0   |
| 0   | 1   | 2   | 3   | 1     | 0     | 0     | $k$ |
| 0   | 3   | -1  | 1   | 0     | 1     | 0     | 45  |
| 0   | 2   | 4   | -1  | 0     | 0     | 1     | 50  |

- (a) Formulate the information from **Fig. 4.1** as a linear programming problem, stating the objective and listing all the constraints as simplified inequalities with integer coefficients. [3]

It is given that  $k$  is a constant where  $15 < k < 135$ .

- (b) Perform **one** iteration of the simplex method, using an entry in the  $x$  column as the pivot element. Give each entry as an exact value. [3]

The tableau after a second iteration of the simplex method, is given in **Fig. 4.2**.

**Fig. 4.2**

| $P$ | $x$ | $y$            | $z$ | $s_1$          | $s_2$          | $s_3$ | RHS                 |
|-----|-----|----------------|-----|----------------|----------------|-------|---------------------|
| 1   | 0   | $\frac{21}{8}$ | 0   | $\frac{1}{8}$  | $\frac{5}{8}$  | 0     | $\frac{k+225}{8}$   |
| 0   | 0   | $\frac{7}{8}$  | 1   | $\frac{3}{8}$  | $-\frac{1}{8}$ | 0     | $\frac{3(k-15)}{8}$ |
| 0   | 1   | $-\frac{5}{8}$ | 0   | $-\frac{1}{8}$ | $\frac{3}{8}$  | 0     | $\frac{-k+135}{8}$  |
| 0   | 0   | $\frac{49}{8}$ | 0   | $\frac{5}{8}$  | $-\frac{7}{8}$ | 1     | $\frac{5(k+17)}{8}$ |

- (c) Explain how the tableau in **Fig. 4.2** shows that the solution obtained after a second iteration is optimal. [1]
- (d) Given that the optimal value of  $P$  is 36, determine the corresponding value of each basic variable after the second iteration. [3]

## 6

- 5 The objective function for a LP formulation to find the shortest path from A to G in a network with seven vertices, A, B, ..., G is given below.

$$\text{Minimise } 5AB + 13AC + 8AD + 6BC + 6CB + 14BF + 14FB + 12BE + 12EB + 6DE + 6ED + 6CF + 6FC + 2EF + 2FE + 22DG + 11EG + 7FG$$

- (a) Explain why both BC and CB appear in the objective function. [1]

One of the constraints of the LP formulation is

$$AD + ED - DE - DG = 0$$

- (b) Explain the purpose of this constraint in the LP formulation. [1]

- (c) Using the vertices given in the Printed Answer Booklet draw a network with **12** arcs to represent the information given in the above objective function. [2]

- (d) (i) Apply Dijkstra's algorithm to the network drawn in part (c), to find the weight of the shortest path from A to G. [5]

- (ii) Write down the shortest path from A to G. [1]

- (e) A computer takes 0.014 seconds to solve a shortest path problem on a network with seven vertices using Dijkstra's algorithm. Approximately how long, in seconds, will it take the same computer to solve a shortest path problem on a network with seven hundred vertices using Dijkstra's algorithm? [2]

- (f) (i) Apply Prim's algorithm, starting at A, to find the minimum spanning tree for the network drawn in part (c). You must, as part of your solution, state the order in which the arcs were included in the tree. [2]

- (ii) State the weight of the minimum spanning tree. [1]

- 6 Each year, to raise money for charity, Henry makes and sells three different sizes of T-shirt, small, medium, and large.

This year, the number of T-shirts that Henry plans to make are subject to the following constraints.

Each small T-shirt requires 25 minutes to make, each medium T-shirt requires 40 minutes to make, and each large T-shirt requires 60 minutes to make.

The total time that Henry can spend making all the T-shirts must not exceed 100 hours.

From his experience in previous years, Henry decides to make at most 50 medium T-shirts and at least 100 T-shirts in total.

Furthermore, Henry decides to make small and large T-shirts in the ratio of 2 small T-shirts for every 5 large T-shirts.

Henry plans on selling each small T-shirt for £5, each medium T-shirt for £6 and each large T-shirt for £8. Henry wants to maximise the total income from the sale of the T-shirts. Total income is the money made from selling T-shirts, ignoring the cost of materials.

Let  $x$ ,  $y$  and  $z$  represent the number of small T-shirts, medium T-shirts, and large T-shirts respectively that Henry makes.

- (a) By representing the feasible region for  $x$  and  $y$  graphically, determine how many of each size of T-shirt Henry should make to maximise the total income from selling the T-shirts. You must show all your working. [10]
- (b) Using the solution found in part (a), find the maximum possible total income. [1]
- (c) Give a reason why the total income found in part (b) may not be achieved. [1]

**END OF QUESTION PAPER**

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